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Eccentricity pumping of a planet on an inclined orbit by a disc

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ABSTRACT

In this paper, we show that the eccentricity of a planet on an inclined orbit with respect to a disc can be pumped up to high values by the gravitational potential of the disc, even when the orbit of the planet crosses the disc plane. This process is an extension of the Kozai effect. If the orbit of the planet is well inside the disc inner cavity, the process is formally identical to the classical Kozai effect. If the planet's orbit crosses the disc but most of the disc mass is beyond the orbit, the eccentricity of the planet grows when the initial angle between the orbit and the disc is larger than some critical value which may be significantly smaller than the classical value of 39 degrees. Both the eccentricity and the inclination angle then vary periodically with time. When the period of the oscillations of the eccentricity is smaller than the disc lifetime, the planet may be left on an eccentric orbit as the disc dissipates.

Key words: celestial mechanics — planetary systems — planetary systems: formation — planetary systems: protoplanetary discs — planets and satellites: general

1 INTRODUCTION

Among the 240 extrasolar planets that have been detected so far with a semi–major axis larger than 0.1 astronomical unit (au), about 100 have an eccentricity $e > 0.3$. Five of them even have $e > 0.8$. Such large eccentricities, which cannot be the result of disc–planet interaction (Papaloizou et al. 2001), are probably produced by planet–planet interactions, either through scattering or secular perturbation (see Ford & Rasio 2008 and references therein), that occur after the disc dissipates (Juric & Tremaine 2008, Chatterjee et al. 2008, Ford & Rasio 2008).

Here, we show that high eccentricities can be pumped *by the disc* if the orbit of the planet is inclined with respect to the disc. The process involved is an extension of the Kozai mechanism, in which a planet is perturbed by a distant companion on an inclined orbit (Kozai 1962). While the Kozai effect has always been studied for the case in which the companion is far away from the planet, the process investigated here is shown to be efficient even if the orbit of the planet crosses the disc. The classical Kozai effect has of course been very well studied. Here, we show that some significant differences occur when the classical scenario is extended to apply to a disc.

In section 2 we review the Kozai effect, and show that the same behaviour is expected whether the planet is perturbed by a distant companion or by a ring of material orbiting far away. In section 3, we present the results of numerical simulations of the interaction between a planet on an inclined orbit and a disc. We show that, provided most of the mass in the disc is beyond the orbit, and the initial inclination is larger than some critical value, the gravitational potential from the disc causes the eccentricity and the inclination of the planet's orbit to oscillate with time. This may occur even if the orbit crosses the disc. In section 4 we summarise our findings, and discuss under which conditions this mechanism could operate. The important result is that a planet on an inclined orbit with respect to the disc and located in or within the planet formation region may have its eccentricity pumped up to high values by the interaction with the disc. This is of astronomical interest, since inclinations are beginning to be measured for extrasolar planets.

2 REVIEW OF THE KOZAI EFFECT AND EXTENSION TO A DISC

We consider a planet of mass M_p orbiting around a star of mass $M_\star$ which is itself surrounded by a ring of material of mass $M_{\rm disc}$. The ring is in the equatorial plane of the star whereas the orbit of the planet is inclined with respect to this plane. The motion of the planet is dominated by the star, so that its orbit is an ellipse slightly perturbed by the gravitational potential of the ring. We study the secular

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perturbation of the orbit due to the ring. We denote by (X, Y, Z) the Cartesian coordinate system centred on the star and (r, φ, θ) the associated spherical coordinates. The ring is in the (X, Y)–plane between the radii R_i and $R_o > R_i$. We suppose that the angular momentum of the disc is large compared to that of the planet's orbit so that the effect of the planet on the disc is negligible: the disc does not precess and its orientation is invariable. The gravitational potential exerted by the ring at the location of the planet is:

$$\Phi = -G \int_{R_i}^{R_o} \Sigma(r) r dr \int_0^{2\pi} \frac{d\alpha}{\left(r^2 + r_p^2 - 2 r r_p \cos\alpha \sin\theta_p\right)^{1/2}}, \quad (1)$$

where the subscript p refers to the planet and $\Sigma(r)$ is the mass density in the ring. We assume:

$$\Sigma(r) = \Sigma_0 \left(\frac{r}{R_o} \right)^{-n}, \quad (2)$$

where:

$$\Sigma_0 = \frac{(-n+2) M_{\rm disc}}{2\left(1-\eta^{-n+2}\right) \pi R_o^2}, \quad (3)$$

with $\eta \equiv R_i/R_o$. We suppose that $R_i \gg r_p$, so that the square root in equation (1) can be expanded in r_p/r and integrated to give:

$$\Phi = - \frac{-n+2}{1-\eta^{-n+2}} \frac{G M_{\rm disc}}{R_o} \left[\frac{1-\eta^{1-n}}{1-n} + \right. \quad (4)$$

$$\left. \frac{-1+\eta^{-1-n}}{1+n} \frac{r_p^2}{2R_o^2} \left(-1 + \frac{3}{2} \sin^2\theta_p \right) \right]. \quad (5)$$

In the classical Kozai effect, the planet is perturbed by a distant companion of mass M. If we assume the orbit of this outer companion is circular of radius $R \gg r_p$ and lies in the (X, Y)–plane, then the potential averaged over time it exerts at the location (r_p, θ_p) is:

$$\Phi_{\rm Kozai} = - \frac{GM}{R} \left[1 + \frac{r_p^2}{2R^2} \left(-1 + \frac{3}{2} \sin^2\theta_p \right) \right]. \quad (6)$$

Because r_p and θ_p appears in exactly the same way in Φ and $\Phi_{\rm Kozai}$, the secular perturbation on the inner planet, obtained by averaging over the mean anomaly of its orbit, is the same in both cases to within an overall multiplicative factor. The results obtained for the classical Kozai effect can therefore be extended to the case of the disc. In particular, the perturbation due to the disc makes the eccentricity e of the planet to oscillate with time if the initial inclination angle I_0 between the orbit of the planet and the plane of the disc is larger than a critical angle I_c given by:

$$\cos^2 I_c = \frac{3}{5}. \quad (7)$$

The maximum value reached by the eccentricity is then (Innanen et al. 1997):

$$e_{\rm max} = \left(1 - \frac{5}{3} \cos^2 I_0 \right)^{1/2}, \quad (8)$$

and the time $t_{\rm evol}$ it takes to reach $e_{\rm max}$ starting from e_0 is (Innanen et al. 1997):

$$\frac{t_{\rm evol}}{\tau} = 0.42 \left(\sin^2 I_0 - \frac{2}{5} \right)^{-1/2} \ln \left(\frac{e_{\rm max}}{e_0} \right), \quad (9)$$

with the time τ defined as:

$$\tau = \frac{(1+n)(1-\eta^{-n+2})}{(-n+2)(-1+\eta^{-n-1})} \frac{R_o^3 M_\star}{a^3 M_{\rm disc}} \frac{T}{2\pi}, \quad (10)$$

where T is the orbital period of the planet and a is its semi–major axis. Note that the function $\left(\sin^2 I_0 - 2/5\right)^{-1/2}$ decreases very sharply from infinity to ~ 3 as I_0 increases from I_c to about 45° and then decreases by about 50% as I_0 continues to increase up to 90°.

The Z–component of the angular momentum of the orbit, $L_z \propto \sqrt{1-e^2} \cos I$, where I is the inclination angle between the orbit and the plane of the disc, is constant. Therefore I also oscillates with time and is out of phase with e.

3 NUMERICAL SIMULATIONS

We consider a star of mass $M_\star = 1$ M$_\odot$ surrounded by a disc of mass $M_{\rm disc}$ and a planet of mass M_p whose orbit is inclined with respect to the disc. The planet interacts gravitationally with the star and the disc but we take $M_p \ll M_{\rm disc}$ so that it has no effect on the disc. To study the evolution of the system, we use the N–body code described in Papaloizou & Terquem (2001) in which we have added the gravitational force exerted by the disc onto the planet.

The equation of motion for the planet is:

$$\frac{d^2 \mathbf{r}}{dt^2} = - \frac{G M_\star \mathbf{r}}{|\mathbf{r}|^3} - \boldsymbol{\nabla} \Phi - \frac{G M_p \mathbf{r}}{|\mathbf{r}|^3} + \boldsymbol{\Gamma}_{t,r}\ , \quad (11)$$

where $\mathbf{r}$ is the position vector of the planet and Φ is the gravitational potential of the disc given by equation (1) with R_i and R_o being the inner and outer radii of the disc. The third term on the right–hand side is the acceleration of the coordinate system based on the central star. Tides raised by the star in the planet and relativistic effects are included through $\boldsymbol{\Gamma}_{t,r}$, but they are unimportant here as the planet does not approach the star closely. Equation (11) is integrated using the Bulirsch–Stoer method and the integrals involved in $\boldsymbol{\nabla}\Phi$ are calculated with the Romberg method (Press et al. 1993). In most runs, the integration conserves the total energy of the planet and L_Z within 1 to 2%.

The planet is set on a circular orbit at the distance r_p from the star. The initial inclination angle of the orbit with respect to the disc is I_0. In the simulations reported here we have taken $n = 1/2$ in equation (2). The functional form of Σ is shallower than what is usually used for discs, but that has no significant effect on the argument we develop here.

We first compare the numerical results with the analysis summarised in section 2 by setting up a case with $R_i \gg r_p$. In figure 1 we display the evolution of e and I for $M_p = 10^{-3}$ M$_\odot$, $r_p = 1$ au, $M_{\rm disc} = 10^{-2}$ M$_\odot$, $R_o = 100$ au, $R_i = 50$ au and $I_0 = 42^\circ.3$. For this run, L_Z is conserved within 2% but the energy of the planet is conserved only within 10%. We are here in the conditions of the analysis of section 2 with $\eta = 0.5$. From equation (8), we expect $e_{\rm max} = 0.3$, which is a bit smaller than the value of 0.41 found in the simulation. Also the minimum value of I should be $I_c = 39^\circ.2$ and is observed to be $36^\circ.5$. Note that since the energy varies by about 10% in this run, we do not expect exact agreement between the numerical and the analytical results. We observe that the time it takes to reach $e_{\rm max}$ from

the initial conditions is 2.8×10^7 years, which agrees well with $t_{\rm evol}$ given by equation (9) provided we take $e_0 \simeq 2\times10^{-2}$. As mentioned above, $t_{\rm evol}$ becomes very long when I_0 is smaller than 45°. As the disc lifetime is only a few Myr, e would not have time to reach the maximum value in this case, if starting from a very small value.

Figure 2 shows the evolution of e and I for $M_p = 10^{-3}$ M$_\odot$, $r_p = 20$ au, $M_{\rm disc} = 10^{-2}$ M$_\odot$, $R_o = 100$ au, $R_i = 1$ au and $I_0 = 47^\circ.7$ (case A). We see that e oscillates between $e_{\rm min} = 10^{-2}$ and $e_{\rm max} = 0.7$, whereas I oscillates between $I_{\rm min} = 20^\circ.1$ and $I_{\rm max} = I_0$. The values of $e_{\rm min}$, $e_{\rm max}$ and $I_{\rm min}$ differ from those calculated in the analysis in section 2, but this is expected as the condition $r_p \ll R_i$, that was used in the analysis, is not valid here. However, since most of the mass in the disc is in the outer parts, beyond the planet's orbit, the behaviour we get here is similar to that described in the analysis. The period of the oscillations is $T_{\rm osc} = 2.2 \times 10^5$ years.

For comparison, we have run the classical Kozai case, where the disc is replaced by a planet located on a circular orbit in the (X, Y)–plane. This perturbing planet is at a distance R from the central star and has a mass M. We take M to be the same as the value of $M_{\rm disc}$ above, and consider $R = 50$ and 100 au. The evolution of e and I for the inner planet in that case is shown in Figure 3. Equation (8) gives $e_{\rm max} = 0.5$, which is in very good agreement with the values of 0.5 and 0.55 obtained from the numerical simulations for $R = 100$ and 50 au, respectively. The time it takes to reach $e_{\rm max}$ from $e_{\rm min}$, which is $T_{\rm osc}/2$, is given by equation (9) with e_0 being replaced by $e_{\rm min}$ and $\tau = [R^3 M_\star/(a^3 M)]T/(2\pi)$. We get $t_{\rm evol} = 5.5 \times 10^5$ and 8×10^4 years for $R = 100$ au ($e_{\rm min} = 0.03$) and 50 au ($e_{\rm min} = 0.02$), respectively, which is in excellent agreement with the values seen on Figure 3. The minimum angle reached when $R = 50$ au is about 31°, much larger than the value obtained when the disc is present. Of course this value would decrease if the perturbing planet were moved closer to the inner planet, but then the oscillations would not be regular anymore, as can already be seen in the case $R = 50$ au.

Figure 4 shows the evolution of e and I for the same values of $M_{\rm disc}$ and I_0 as in case A but with $M_p = 4 \times 10^{-3}$ M$_\odot$, $r_p = 1$ au, $R_o = 50$ au and $R_i = 0.5$ au (case B). Here we have $e_{\rm min} = 5 \times 10^{-2}$, $e_{\rm max} = 0.72$, $I_{\rm min} = 16^\circ.4$, $I_{\rm max} = I_0$ and $T_{\rm osc} = 2.0 \times 10^5$ years, comparable to the value found in the previous case.

On dimensional grounds, $T_{\rm osc}$ is expected to be proportional to τ given by equation (10). We have run case A with different values of $M_{\rm disc}$ and have checked that $T_{\rm osc} \propto 1/M_{\rm disc}$. The values between which e and I oscillate though do not depend on $M_{\rm disc}$, as expected from the analysis.

We have run case A with different values of r_p ranging from 2 to 50 au. We observe that for r_p roughly below 10 au, $T_{\rm osc}$ decreases when r_p increases, as expected from the expression of τ. For larger values of r_p though, $T_{\rm osc}$ does not vary much with r_p, and is $\sim$ 2–3×10^5 years. For $r_p = 10$ and 20 au, the extreme values of e and I are roughly the same if the calculations are started from the same I_0 and e_0, but for $r_p = 50$ au the amplitude of the oscillations is very small. In that case, most of the disc mass if not beyond the planet's orbit, so that the Kozai effect disappears.

Finally, we have checked the effect of varying I_0 in case A. We have found that there is a critical value of I_0, $I_c \sim 30^\circ$, below which eccentricity growth was not observed. However, the fact that $I_{\rm min} = 20^\circ.1$ in case A suggests that I_c may actually be smaller. Since the time it takes for e to grow from very small values when I_0 is close to I_c is very long (see equation [9]), the simulations may not have been run long enough for a growth to be observed. As I_0 increases from I_c to 90°, $e_{\rm max}$ grows from 0 to 1. $T_{\rm osc}$ is not observed to vary significantly with I_0, although the time it takes for the eccentricity to grow from its initial value to $e_{\rm max}$ does depend on I_0.

The simulations reported here suggest that when the planet's orbit crosses the disc, eccentricity growth occurs for significantly smaller initial inclination angles than in the classical Kozai effect.

4 DISCUSSION AND CONCLUSION

We have shown that, when a planet's orbit is inclined with respect to a disc, the gravitational perturbation due to the disc results in the eccentricity and the inclination of the orbit oscillating if: (i) most of the disc mass is beyond the planet's orbit, (ii) the initial inclination angle I_0 is larger than some critical value I_c, which may be significantly smaller than in the classical Kozai effect. In the simulations we have performed, in which $\Sigma \propto r^{-1/2}$ and $M_{\rm disc} \ll M_p$ (so that the effect of the planet on the disc is negligible), oscillations occur as long as the initial planet's distance to the star, r_p, is smaller than about half the disc radius R_o. We expect a smaller critical distance when Σ decreases more rapidly with radius. Note that I_c should be independent of the functional form of Σ as long as most of the disc mass is beyond the planet's orbit. The amplitude of the oscillations depends only on I_0. It is small for $I_0 \simeq I_c$ and becomes large when I_0 is increased. In particular, $e_{\rm max} \rightarrow 1$ as $I_0 \rightarrow 90^\circ$. The oscillations of e and I are 180° out of phase. Their period $T_{\rm osc} \propto 1/M_{\rm disc}$. When $r_p \ll R_o$, $T_{\rm osc}$ decreases as r_p increases. For larger values of r_p, $T_{\rm osc}$ does not vary much with this parameter. In the simulations we have performed, $T_{\rm osc} \sim 10^5$ years. As this is much shorter than the disc lifetime, there is a nonzero probability that the planet is left on a highly eccentric orbit as the disc dissipates.

Note that the process discussed in this paper is not a mere trivial extension of the Kozai effect. Indeed, in the classical Kozai effect, the periodic behaviour of e is obtained because the dependence of the perturbing gravitational potential on the planet's argument of pericentre ω is through a term proportional to $\cos 2\omega$. When the orbit of the planet crosses the disc, the gravitational potential has a very different form, and the behaviour of e is much more difficult to predict.

The effect discussed here could be inhibited if other processes induced a precession of the planet's orbit on timescales shorter than $T_{\rm osc}$. That may happen if other planets are present in the system, or because of dissipative tidal torques exerted by the disc, which have been ignored here (Lubow & Ogilvie 2001). In the latter case, the Kozai effect would then work only for planets orbiting inside the disc inner cavity.

When the planet crosses the disc, loss of energy and angular momentum circularises the orbit and aligns it with the disc plane on a timescale $\sim T M_p/(\Sigma(r_p) R_p^2)$, where R_p is

the planet radius (Syer et al. 1991, Ivanov et al. 1999), which can be smaller than $T_{\rm osc}$ for r_p larger than a few au. Taking this process into account together with the Kozai effect may lead to equilibrium values of e and I. This will be studied in a forthcoming paper. Note that the usual type II migration mechanism that applies to planets orbiting in discs would not be relevant here, as it happens when the planet orbits in a gap that is locked in the disc evolution. In the coplanar case, as the disc spirals toward the central star, it carries along the gap and the planet. When the planet is on an inclined orbit, it may still open up a gap but it is not locked in it, so that it is not being pushed in as the disc spirals in.

The mechanism described here relies on the planet being on an inclined orbit, which could happen as a result of: (i) dynamical relaxation of a population of planets formed through fragmentation of a protostellar envelope around a star surrounded by a disc (Papaloizou & Terquem 2001); (ii) mean motion resonances (Thommes & Lissauer 2003, also Yu & Tremaine 2001) and (iii) gravitational interactions between embryos during the planet formation stage (Levison et al. 1998, Cresswell & Nelson 2008). In all cases, the process that makes the orbits inclined also makes them eccentric. According to the results presented here, the disc could pump the eccentricities up to even larger values.

Measures of the projected angle between the axes of the planet's orbit and the stellar rotation, using the Rossiter–McLaughlin effect, are becoming available. So far, only the system XO–3, which has a ~ 12 Jupiter masses planet on a 3.19 days orbit with $e = 0.26$, has been shown to have a spin–orbit misalignment of at least $37^\circ.3$ (Hébrard et al. 2008, Winn et al. 2009). Misalignment has also been reported for the system HD 80606, which has a ~ 4 Jupiter mass planet on a 111.44 days orbit with $e = 0.93$ (Moutou et al. 2009, Pont et al. 2009). As HD80606 is a component of a binary system, the classical Kozai effect could be responsible for the misalignment in this system.

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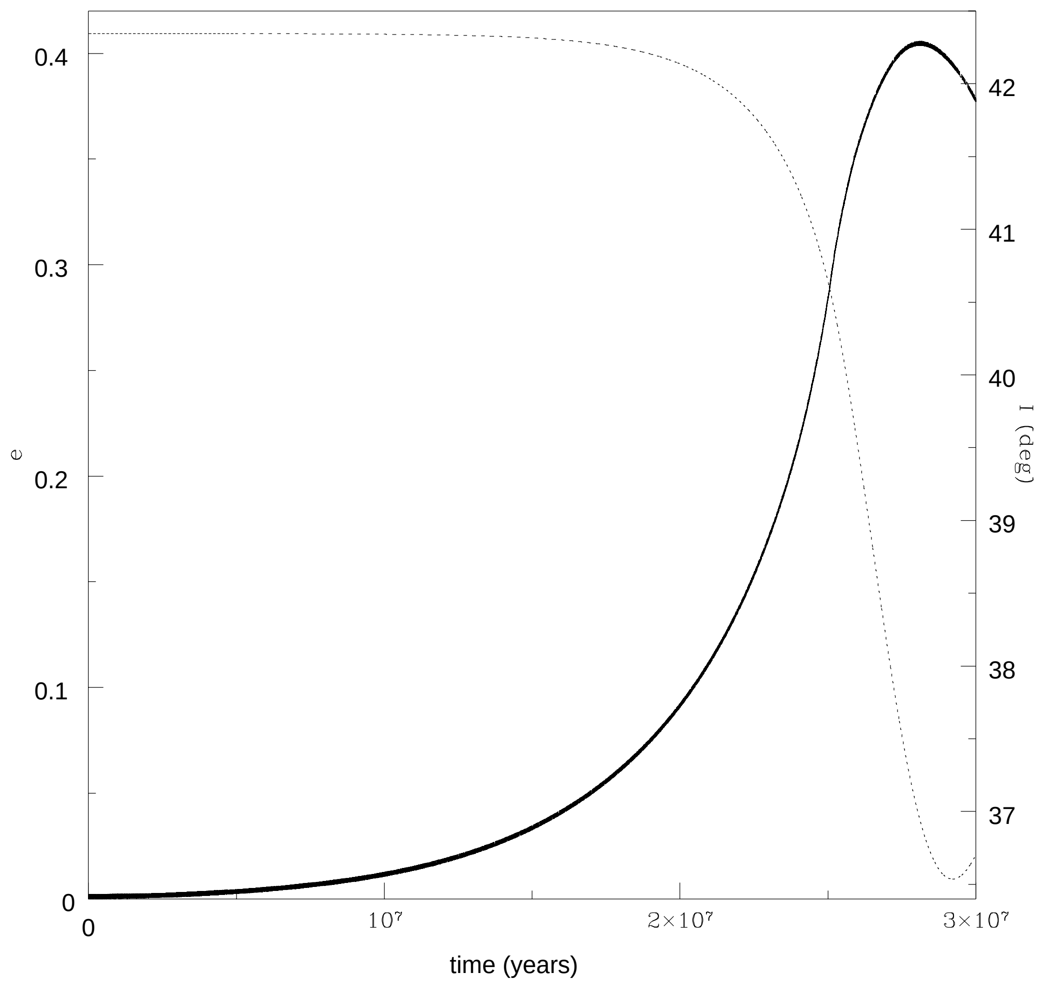

Figure 1. Eccentricity e (*solid line*) and inclination angle I (in degrees, *dotted line*) versus time (in years) for $M_p = 10^{-3}$ $M_\odot$, $r_p = 1$ au, $M_{\rm disc} = 10^{-2}$ $M_\odot$, $R_o = 100$ au, $R_i = 50$ au and $I_0 = 42^\circ.3$.

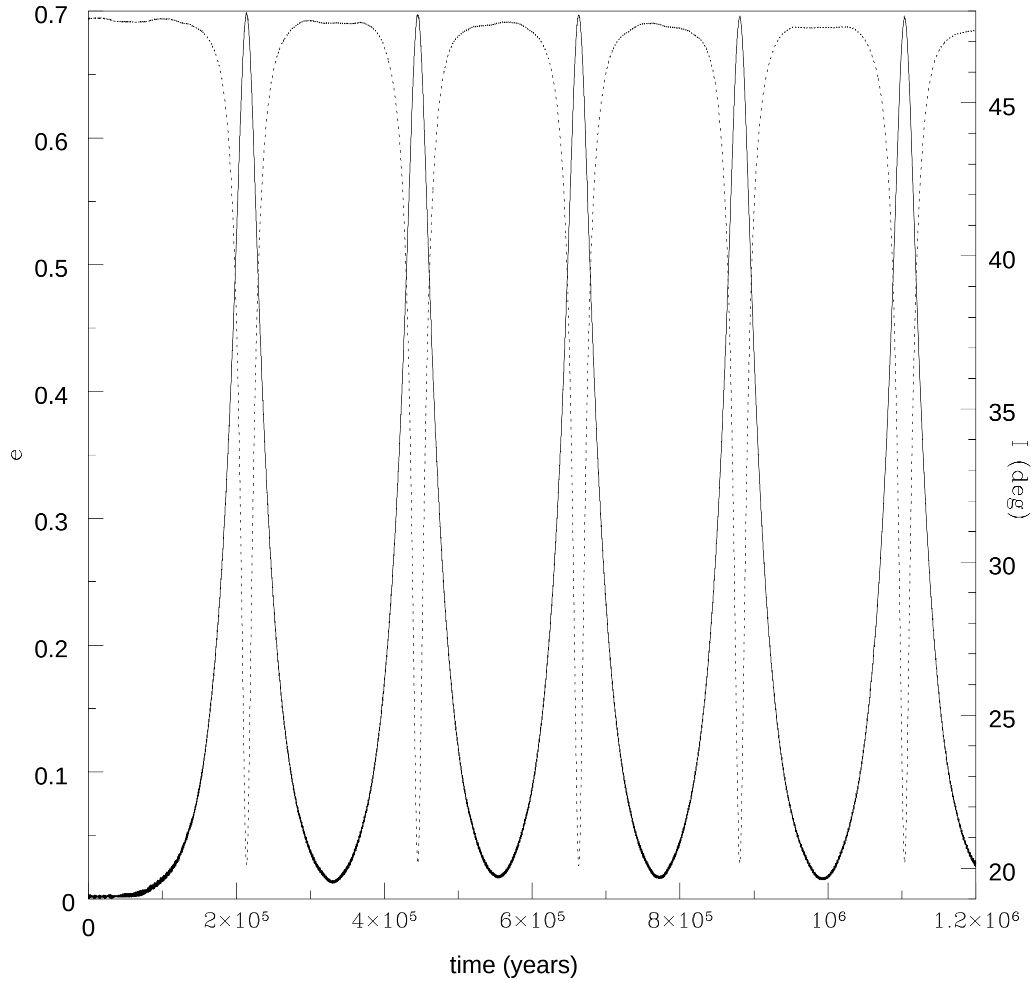

Figure 2. Same as figure 1 but for $r_p = 20$ au, $R_i = 1$ au and $I_0 = 47°.7$.

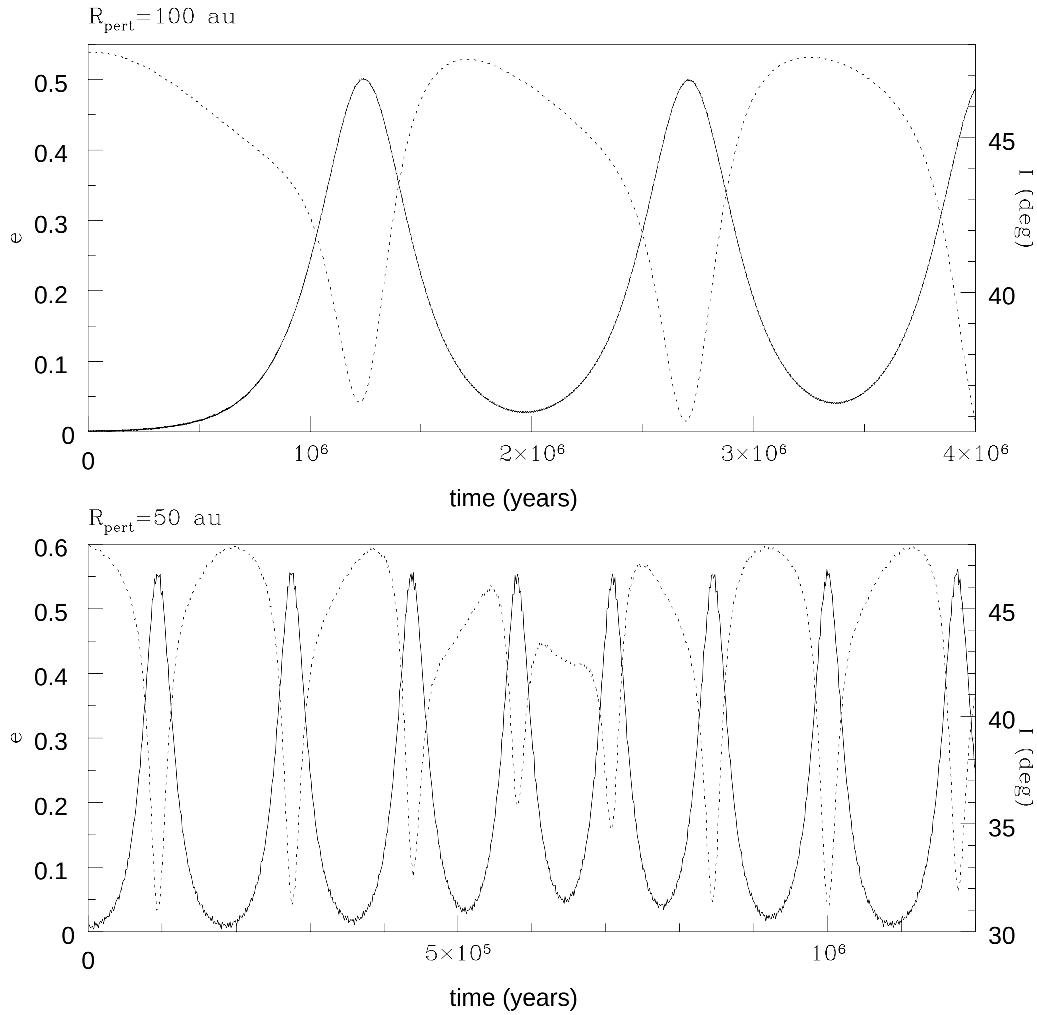

Figure 3. Classical Kozai effect: Same as figure 2 but with the disc being replaced by a planet of mass $M = 10^{-2}$ $M_{\odot}$ and located at a distance $R = 100$ au (*upper plot*) and 50 au (*lower plot*) from the star.

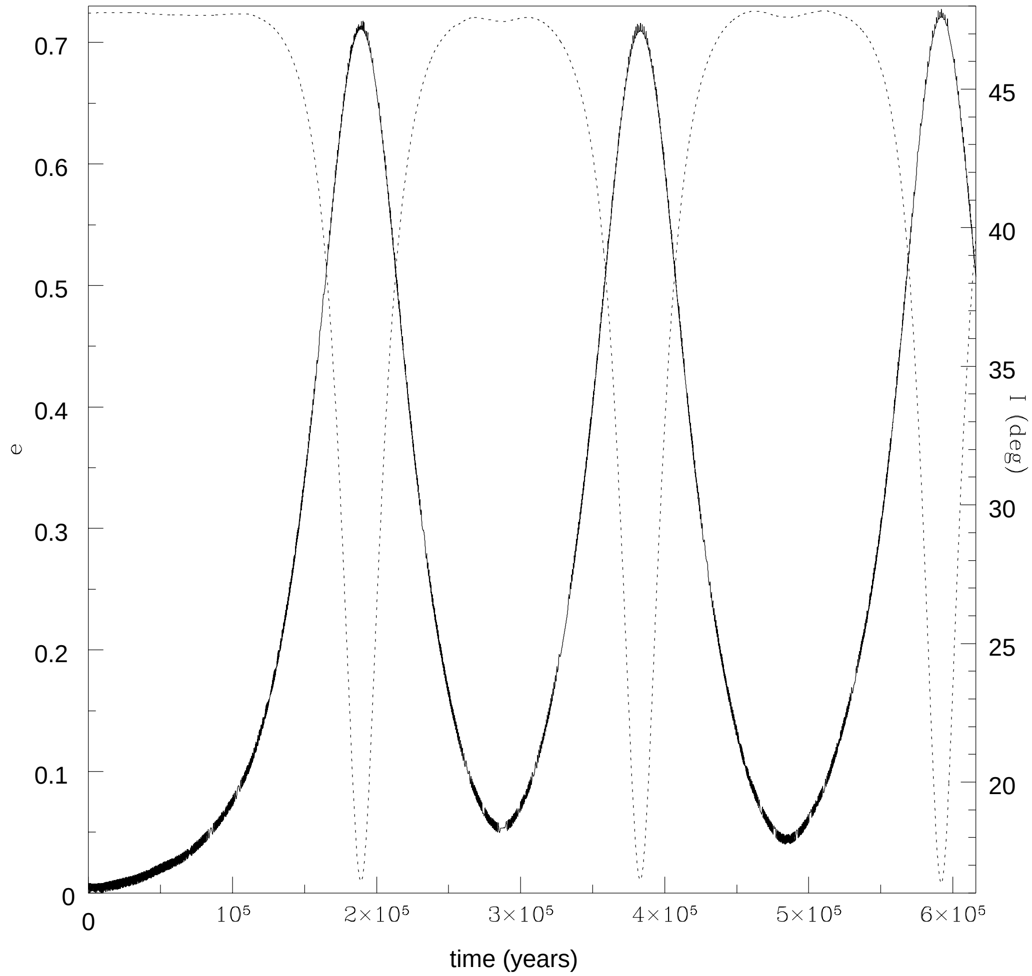

Figure 4. Same as figure 2 but for $M_p = 4 \times 10^{-3}$ $\mathrm{M}_\odot$, $r_p = 1$ au, $R_o = 50$ au, $R_i = 0.5$ au.